Government of Western Australia
School Curriculum and Standards Authority

MATHEMATICS METHODS

Unit 1 and Unit 2

Formula Sheet

(*For use with Year 11 examinations and response tasks*)

2015/75002v2

This document is valid for teaching and examining from 1 July 2015.

Measurement

Circle: $C = 2\pi r = \pi D$, where C is the circumference,
r is the radius and D is the diameter
$A = \pi r^2$, where A is the area

Triangle: $A = \frac{1}{2}bh$, where b is the base and h is the perpendicular height

Parallelogram: $A = bh$

Trapezium: $A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides

Prism: $V = Ah$, where V is the volume and A is the area of the base

Pyramid: $V = \frac{1}{3}Ah$

Cylinder: $S = 2\pi rh + 2\pi r^2$, where S is the total surface area
$V = \pi r^2 h$

Cone: $S = \pi rs + \pi r^2$, where s is the slant height
$V = \frac{1}{3}\pi r^2 h$

Sphere: $S = 4\pi r^2$
$V = \frac{4}{3}\pi r^3$

Functions and graphs

Lines and Linear relationships

For points $P(x_1, y_1)$ and $Q(x_2, y_2)$

Mid-point of P and Q: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Gradient of the line through P and Q: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of the line through P with slope m: $y - y_1 = m(x - x_1)$

Parallel lines: $m_1 = m_2$

Perpendicular lines: $m_1 m_2 = -1$

General equation of a line: $ax + by + c = 0$ or $y = mx + c$

Quadratic relationships

For the general quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$

Completing the square: $ax^2 + bx + c = a\left(x + \frac{b}{2a} \right)^2 + \left(c - \frac{b^2}{4a} \right)$

Discriminant: $\Delta = b^2 - 4ac$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Graphs and Relations

Equation of a circle: $(x - a)^2 + (y - b)^2 = r^2$

where, (a, b) is the centre and r is the radius

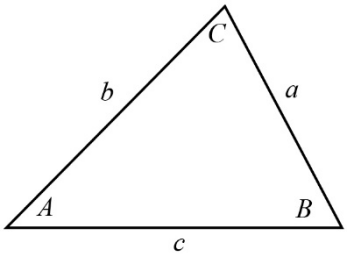
Trigonometric functions

Cosine and sine rules

For any triangle ABC with corresponding length of sides a, b, c

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$



Area of Δ: $A = \frac{1}{2}ab\sin C$

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{1}{2}(a + b + c)$

Circular measure and radian measure

In a circle of radius r, for an arc subtending angle θ (radians) at the centre

Length of arc:	$\ell = r\theta$	Length of chord:	$l = 2r\sin\frac{1}{2}\theta$
Area of sector:	$A = \frac{1}{2}r^2\theta$	Area of segments:	$A = \frac{1}{2}r^2(\theta - \sin\theta)$

Trigonometric functions: (fundamentals)

$$\sin(-\theta) = -\sin\theta \qquad \cos(-\theta) = \cos\theta \qquad \tan(-\theta) = -\tan\theta$$

$$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta \qquad \cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta \qquad \sin^2\theta + \cos^2\theta = 1$$

Angle sum and difference identites

$$\sin(A \pm B) = \sin A\ \cos B \pm \cos A\ \sin B$$
$$\cos(A \pm B) = \cos A\ \cos B \mp \sin A\ \sin B$$

Counting and probability

Combinations

Number of combinations:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(of r objects taken from a set of n distinct objects)

Binomial expansion:

$$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + \cdots + \binom{n}{r}x^{n-r}y^r + \cdots + y^n$$

Binomial coefficients:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n\times(n-1)\times\cdots\times(n-r+1)}{r\times(r-1)\times\cdots\times 2\times 1}$$

Probability

Fundamentals of probability:

$$P(\text{complement of } A) = P(\overline{A}) = 1 - P(A)$$

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B \mid A)$$
$$= P(B)P(A \mid B)$$

Conditional probability:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \text{ for } P(A) \neq 0$$

Exponential functions

Index laws:

For a, $b > 0$ and m,n real,

$$a^m b^m = (ab)^m \qquad a^m a^n = a^{m+n} \qquad (a^m)^n = a^{mn}$$

$$a^{-m} = \frac{1}{a^m} \qquad \frac{a^m}{a^n} = a^{m-n} \qquad a^0 = 1$$

For $a > 0$, m an integer and n a positive integer, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Arithmetic and geometric sequences and series

Arithmetic sequences

For initial term a and common difference d:

$$T_n = a + (n-1)d, n \geq 1$$

$$T_{n+1} = T_n + d, \text{where } T_1 = a$$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

Geometric sequences

For initial term a and common ratio r:

$$T_{n+1} = rT_n, \text{where } T_1 = a$$

$$T_n = ar^{n-1}, n \geq 1$$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S_\infty = \frac{a}{1-r}, \ |r| < 1$$

Introduction to differential calculus

Rates of change

Difference quotient: $$\frac{\delta y}{\delta x}=\frac{f\left(x+h\right)-f\left(x\right)}{h}$$

Derivative (concept): $$\frac{dy}{dx}=f'\left(x\right)=\lim_{h\to 0}\frac{f\left(x+h\right)-f\left(x\right)}{h}$$

Computation of derivatives: $$\frac{d}{dx}\left(x^n\right)=nx^{n-1}.$$

Anti-derivatives: $$\text{If } f'\left(x\right)=ax^n \text{ then } f\left(x\right)=\frac{ax^{n+1}}{n+1}+c\left(\text{constant}\right), n\neq -1$$

Note: Any additional formulas identified by the examination writers as necessary will be included in the body of the particular question.